Application of Müller EDT formulae to 4πβ(LS)-γ coincidence counting

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ICRM

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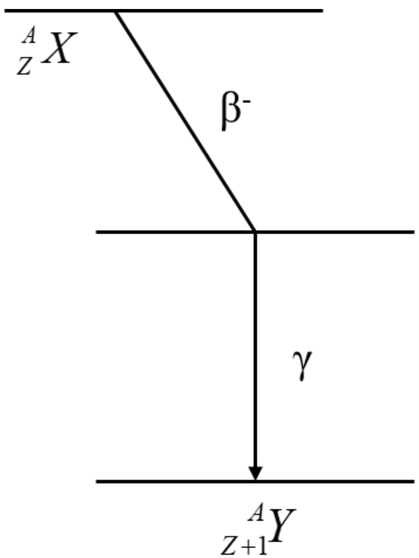
Contents

1. Introduction

Basic principle of 4πβ-γ coincidence

The basic principle of coincidence counting can be described through the ideal case with a simple decay-scheme. In this case, the activity A can be calculated using an expression which depends only on the counting rates in the β- and γ-channels, Nβ and Nγ, and the coincidence counting rate Nc:



$$N_{\beta} = N_0 \cdot \varepsilon_{\beta}$$

$$N_{\gamma} = N_0 \cdot \varepsilon_{\gamma}$$

$$N_{c} = N_0 \cdot \varepsilon_{\beta} \cdot \varepsilon_{\gamma}$$

$$N_0 = \frac{N_{\beta} \cdot N_{\gamma}}{N_c}$$

Correction formulae for coincidence counting

- **In practice**

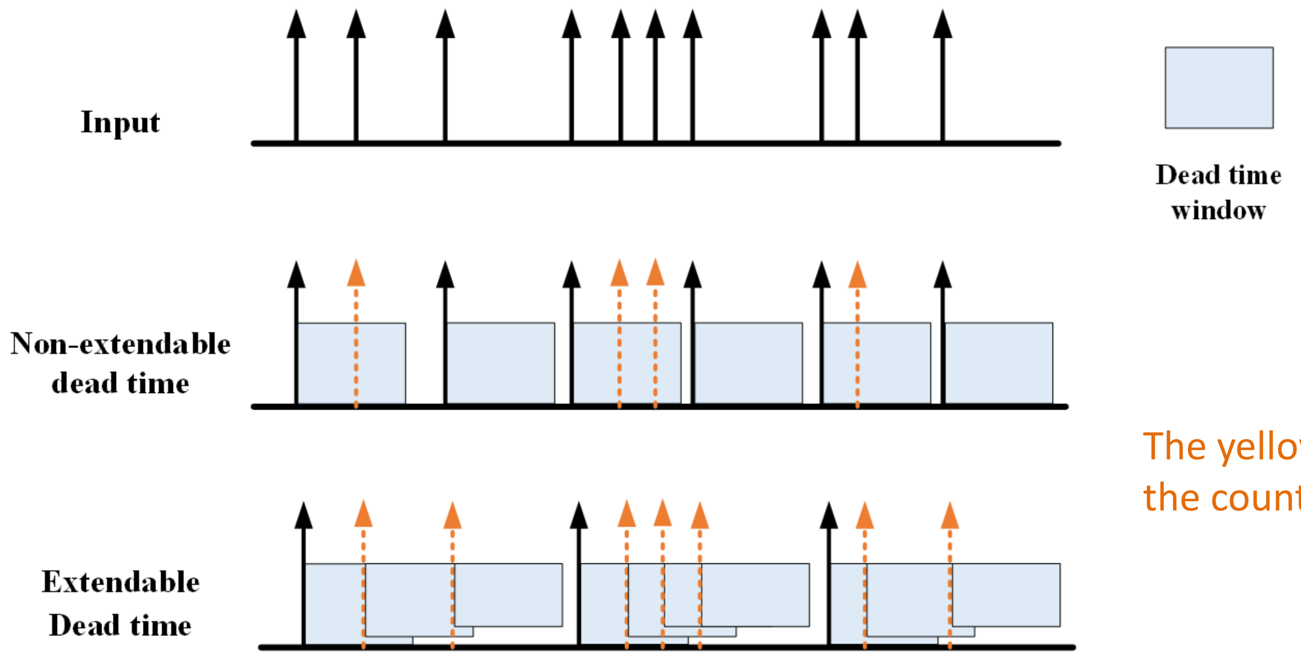
Coincidence measurement may affected by many factors such as "background", "dead time", "accidental coincidences", "β detector sensitivity to γ rays" and so on, which requires a series of complex corrections.

Dead-time counting losses ⟺ all observable count rates are affected by losses due to dead time in detectors and electronic circuits

Accidental coincidences ⟺ a resolving time has to be introduced in order to avoid any loss of genuine coincidences; this, in turn, leads to the recording of accidental coincidences

Correction formulae for coincidence counting

- **Two types of dead time: Non-Extendable & Extendable**



The yellow arrow indicates the counting loss

The difference between the two types of dead time is that Non-Extendable dead time is only triggered by every event that is recorded, whereas Extendable dead time is riggered by all input events.

1. Introduction

Correction formulae for coincidence counting

① Formulae for **Non-Extendable** Dead Times

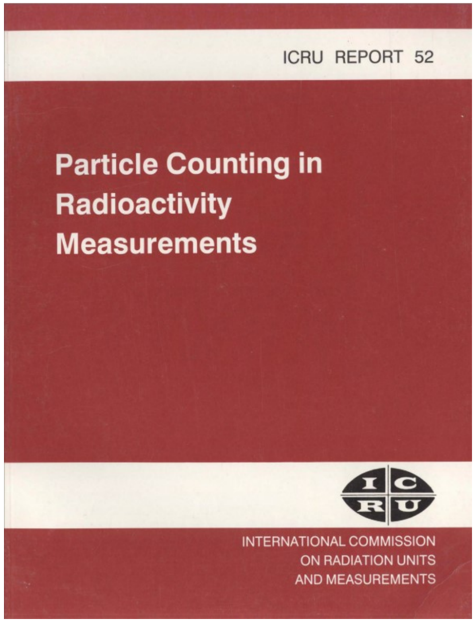
The exact formulation to correct coincidence counting data for the effects of non-extendable dead time and accidental coincidences was discovered by Cox and Isham(1977) and developed by Smith (1978;1979; 1987; 1988) to give exact formulae and a simple-to-use high-order approximation.

- **Non-Extendable dead times must be imposed to both channels**

② Formulae for **Extendable** Dead Times

The exact Formulae to correct coincidence counting data for the effects of extendable dead time was developed by Müller (1977)

- **Extendable dead times must be imposed to both channels**

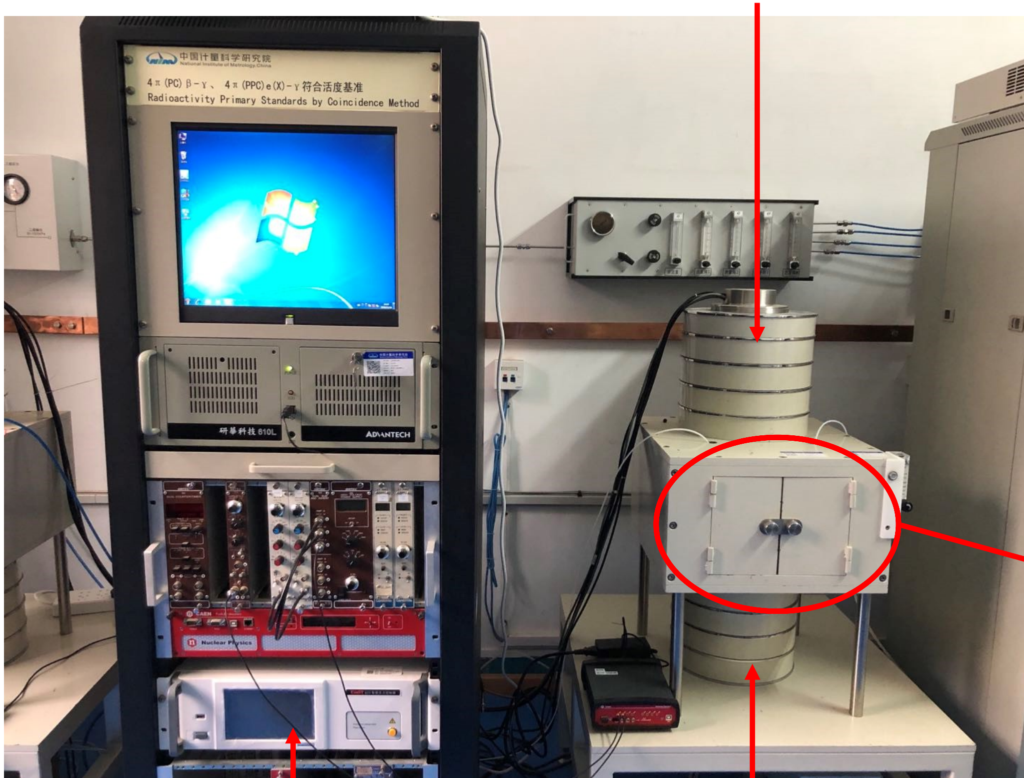


Details can be found in ICRU report 52

2. Coincidence counting system at NIM

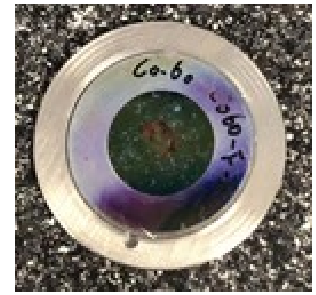
2.1 4πβ(PC)-γ DCC system

NaI(TI)

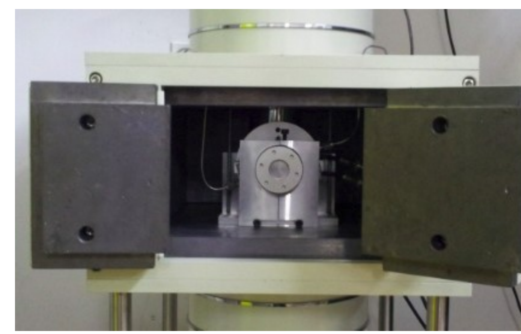


Gas pressure control

NaI(TI)

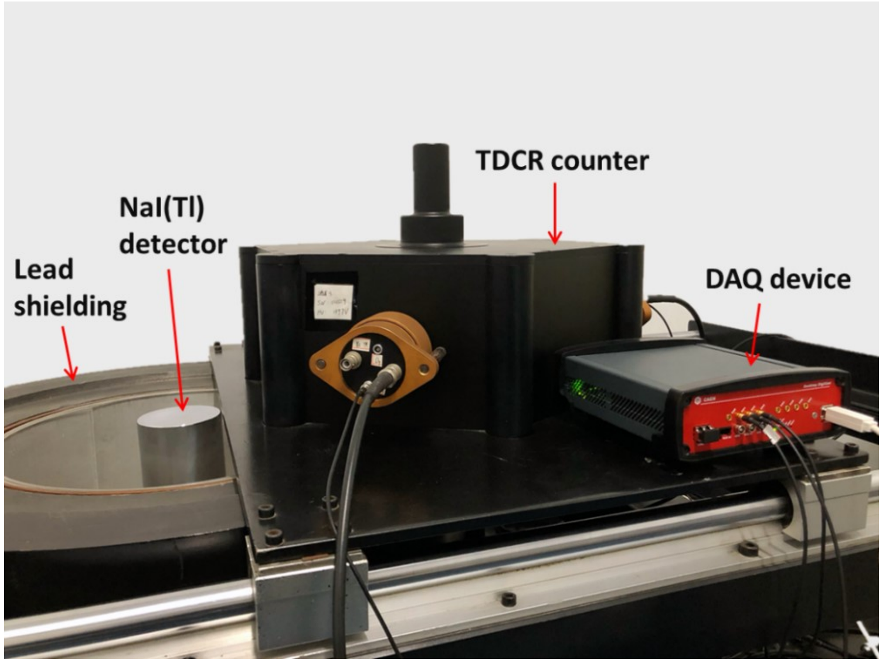


VYNS foil source



Proportional counter

2.2 4πβ(LS)-γ DCC system



- The β-channel is equipped with a custom built TDCR counter, and the γ-channel is obtained by using a 3-inch NaI(Tl) scintillation detector, which is placed perpendicular below the TDCR counter.
- TDCR counter can move left and right on the horizontal plane to facilitate the replacement of the NaI(Tl) scintillation detector.

2.2 4πβ(LS)-γ DCC system

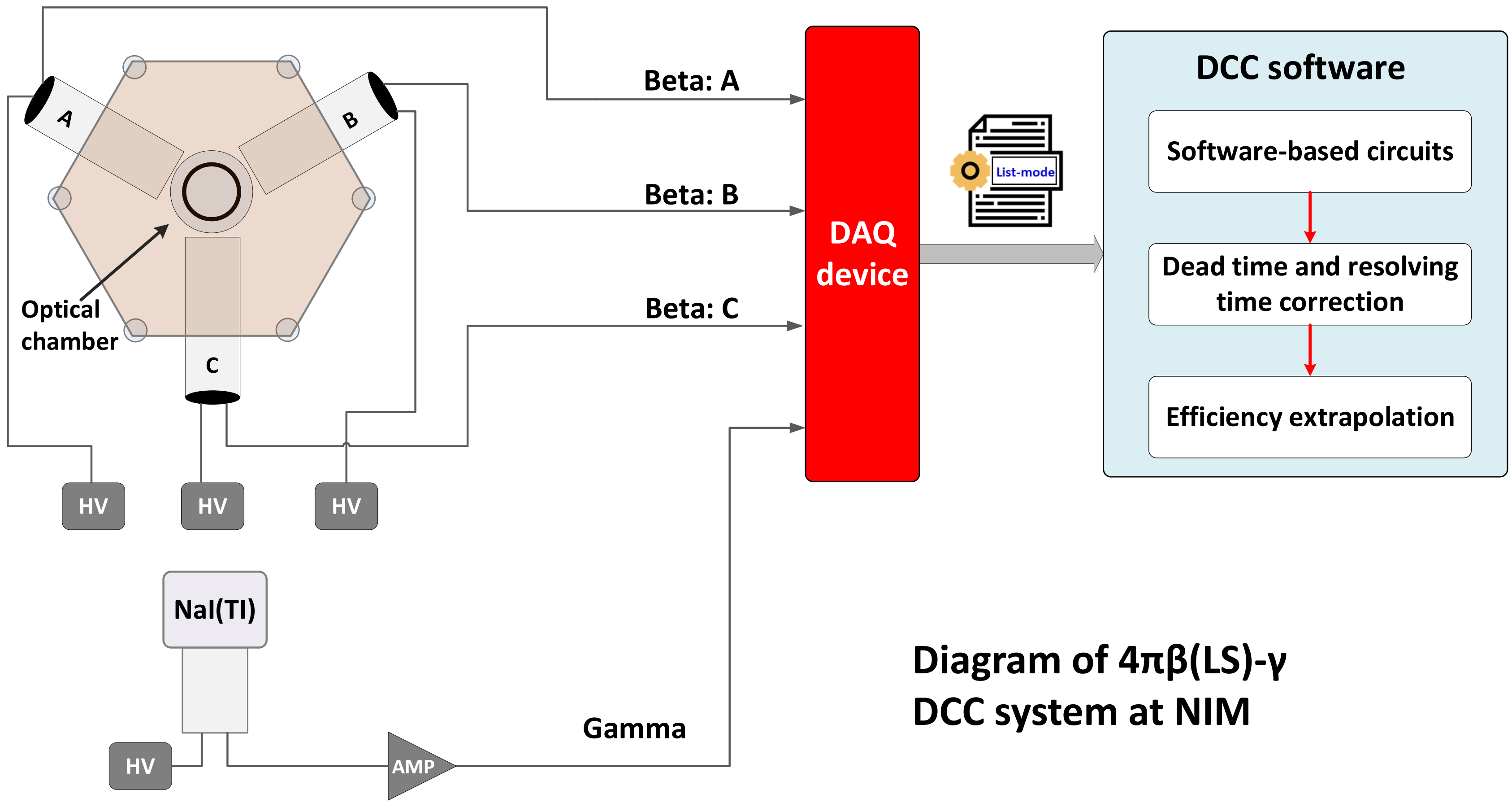
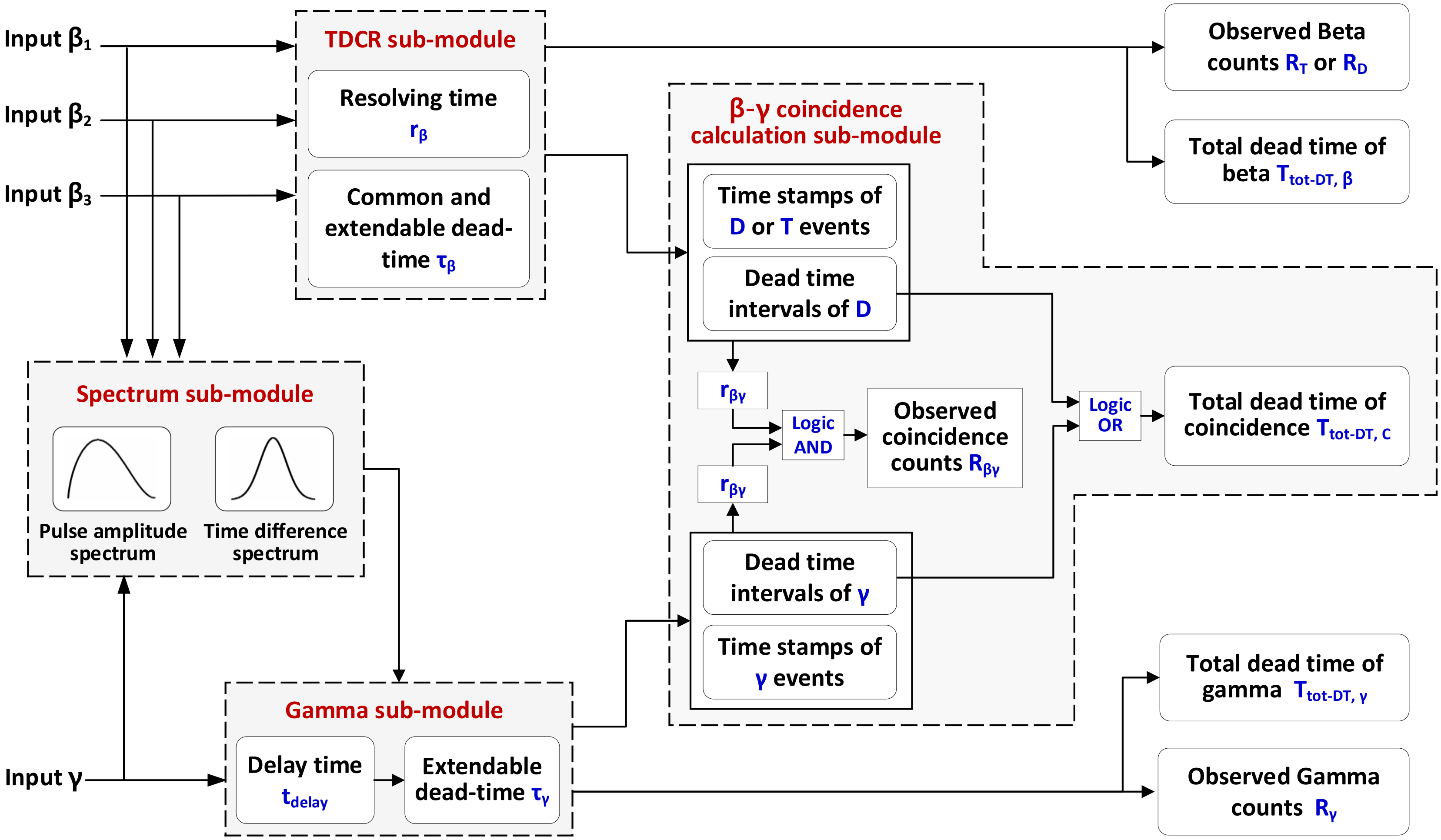


Diagram of 4πβ(LS)-γ DCC system at NIM

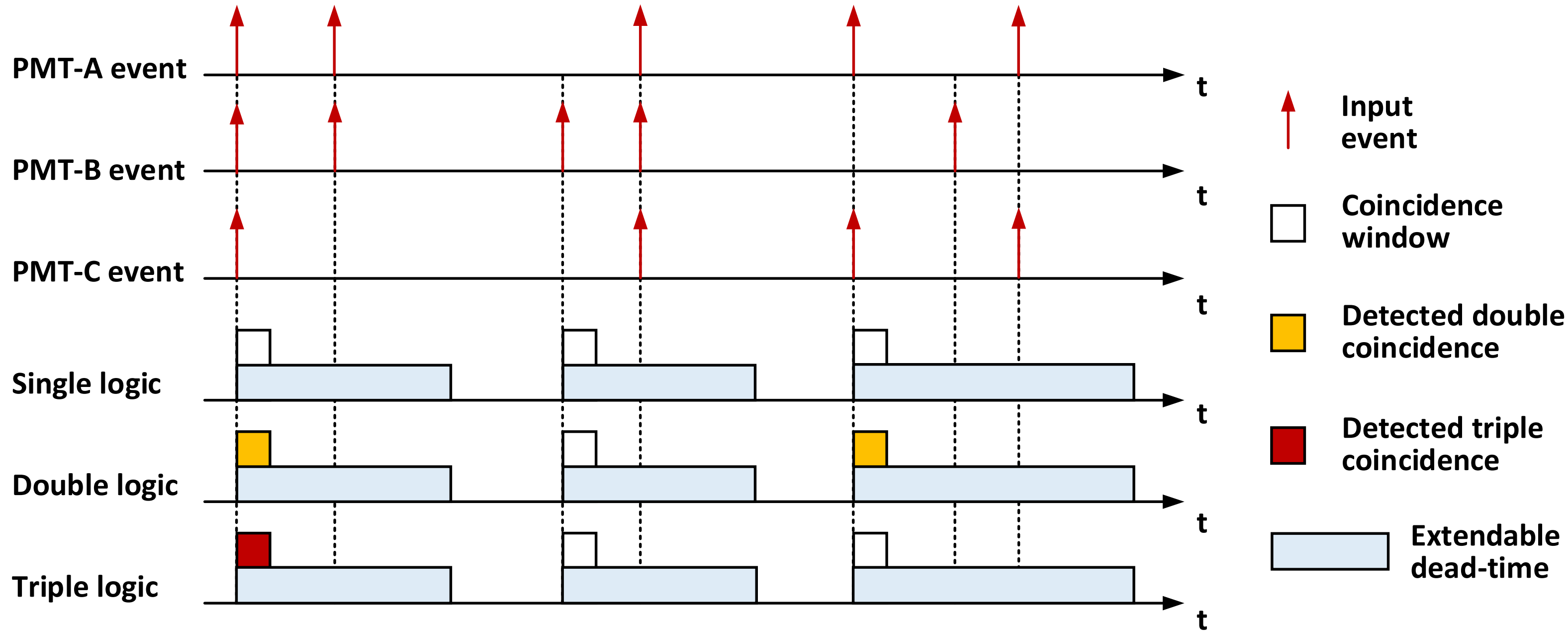
The 4πβ(LS)-γ DCC system is made up of three main parts, namely detection setup, data acquisition (DAQ) unit and corresponding DCC software.

3.1 Software-based circuit module

The "software-based circuit module" is used to realize the circuit functions of imposing delay time, dead-time, coincidence resolving-time, and calculating coincidence etc. The module contains four main sub-modules.



- **TDCR sub-module** to implement TDCR counting model in β-channel



a) The imposed dead time is **extendable** and **common** to all channels, that is, an incoming pulse in any sub-channel(A,B,C) will trigger the dead-time window for all three sub-channels.

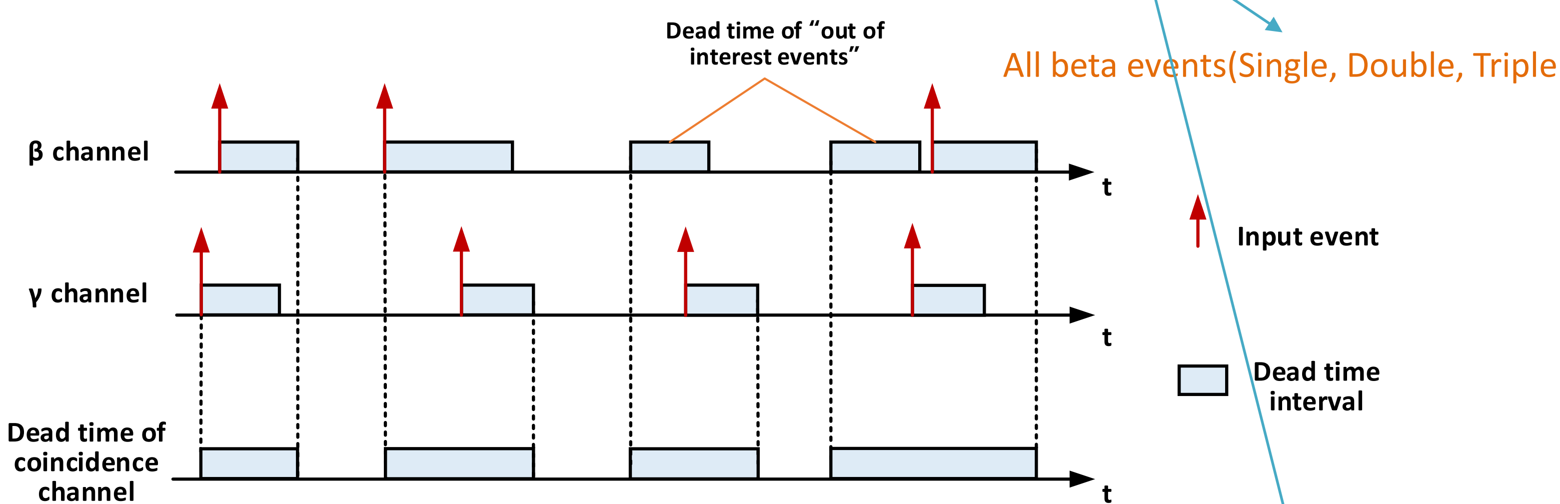
b) Any signals in dead time will only extend dead time, but won't start a new coincidence window.

c) Only the signal not during the dead-time will starts both **dead-time window** and **coincidence window**.

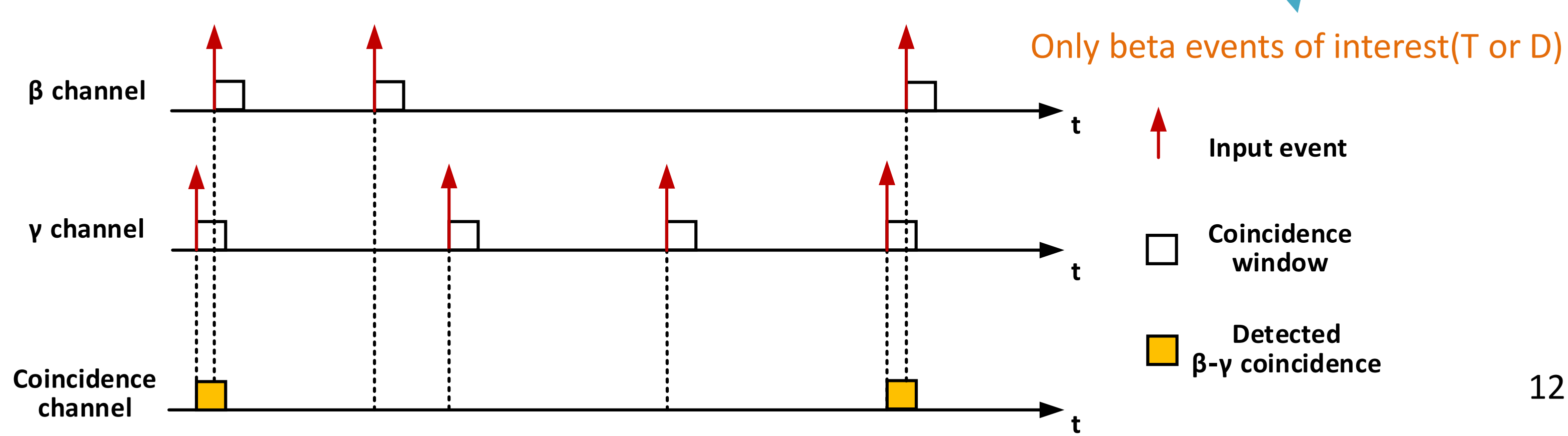
d) The **total dead-time** of β-channel is obtained by the logic OR of the individual dead-time intervals of the three sub-channels.

β-γ coincidence calculation sub-module

In this module, the coincidence process and the dead-time calculation process are treated independently because they involve the participation of different beta events.



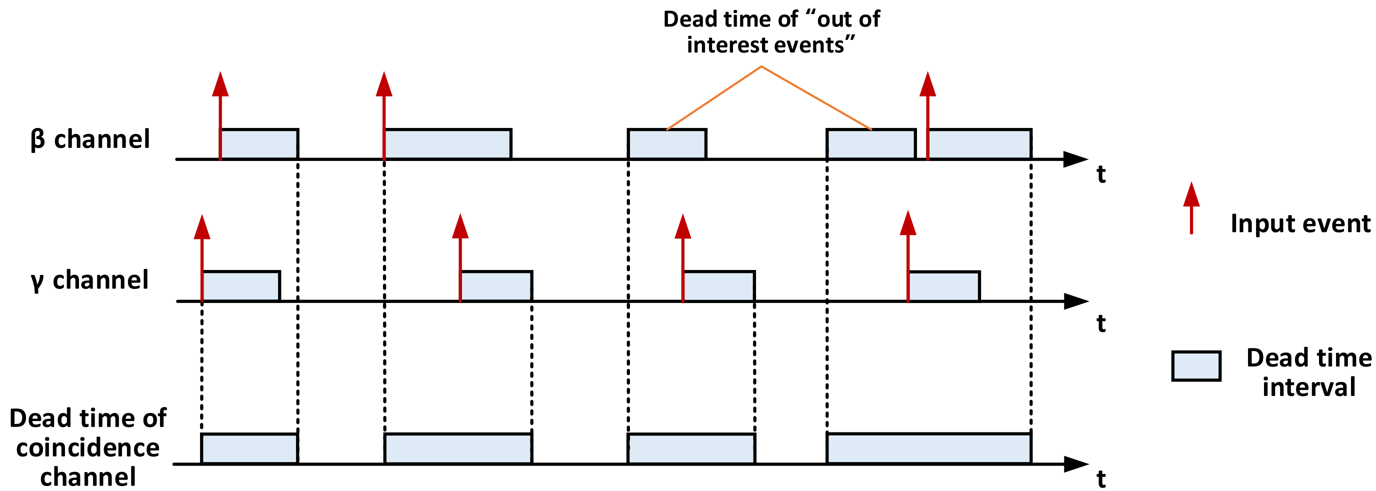
Dead-time logic for coincidence channel



Coincidence logic for β-γ coincidence

● β-γ coincidence calculation sub-module

Dead-time logic for coincidence channel ⇨



a) The coincidence channel will be in the "dead" state, when either the β-channel or γ-channel is in the "dead" state.

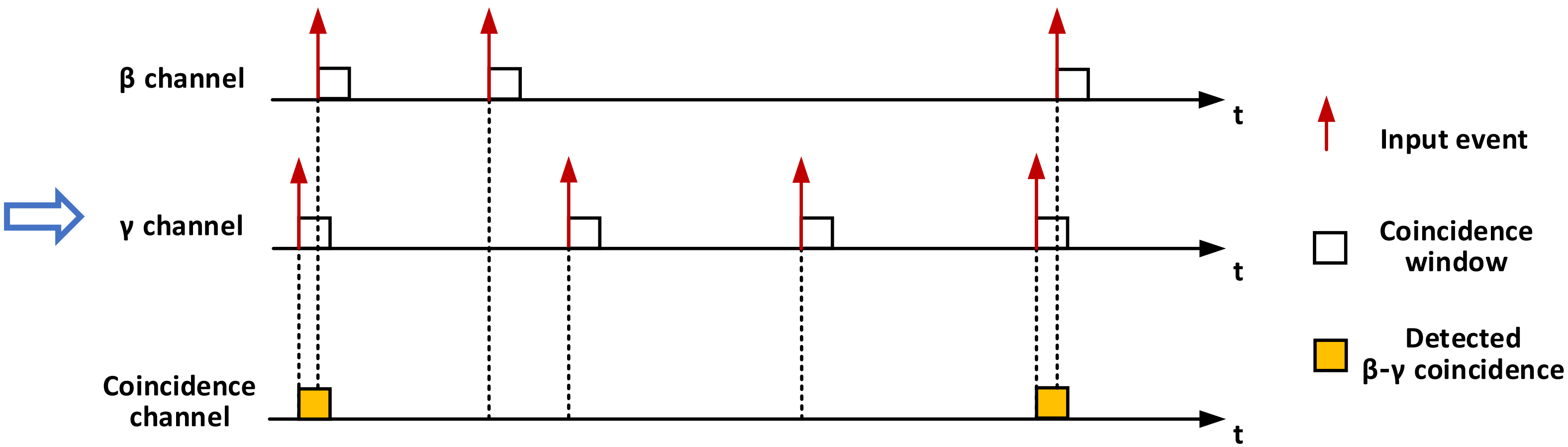
b) The **total dead-time** of coincidence channel is obtained by the **logic OR** of the individual dead-time intervals of the corresponding beta and gamma channels.

c) In particular, there is a special kind of dead-time interval in β-channel, which is triggered by "**out of interest events**". For examples, when triple coincidence events are chosen as the output of β-channel, the non-coincident events and double coincidence events are "out of interest events".

d) The "out of interest events" will only contribute to the total dead-time of coincidence channel, but not to the coincidence calculation.

- β-γ coincidence calculation sub-module

Coincidence logic for β-γ coincidence



a) For any input event from β-channel or γ-channel, it will open a coincidence window, and then search for the follow-up nearest neighbor event.

b) If the event occurs in the same channel, the coincidence will not be registered, and then the former event will be discarded.

c) If the event occurs in another channel, it needs to further determine whether the event falls into the coincidence window. If so, the coincidence channel will register a coincidence and discard these two events. If not, no record of coincidence will be made, and only the former event will be discarded.

3.2 Dead time and resolving time correction

- **γ channel: Dead time correction**

Both exponential correction formula for extendable dead-time and live time correction formula are applicable.

Exponential formula correction: $R_\gamma = \rho_\gamma \cdot e^{-\rho_\gamma \cdot \tau_\gamma}$

Live time correction: $R_\gamma = \rho_\gamma \cdot T_\gamma$ Where T_γ is the live-to-total time ratio in γ-channel

- **β channel: Dead time correction**

Since TDCR method was applied in β channel, in which the common extendable dead-time was imposed, the formula correction is no longer applicable. So the live time correction formula will be applied.

Live time correction: $R_\beta = \rho_\beta \cdot T_\beta$ Where T_β is the live-to-total time ratio in β-channel

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② dead-time and resolving-time correction

- **For coincidence channel**

Müller correction formula

$$\begin{cases} R_c = R_f + R_{\beta\gamma} \\ R_f = \dfrac{R_\beta \rho_g}{\rho_{\beta\gamma}} \left[\exp(-\rho_\gamma \tau_\gamma + \rho_{\beta\gamma} \tau_\beta) \right] \cdot \left[1 - \exp(-\rho_{\beta\gamma} r_\gamma) \right] \\ \qquad + \dfrac{R_\gamma \rho_b}{\rho_{\beta\gamma}} \left[\exp(-\rho_b \tau_\beta) \right] \cdot \begin{Bmatrix} \rho_{\beta\gamma} \left[r_\beta - \left| r_\beta - \tau_\gamma + \tau_\beta \right| \right] \\ +1 - \exp\left(-\rho_{\beta\gamma} \left| r_\beta - \tau_\gamma + \tau_\beta \right| \right) \end{Bmatrix} \\ R_{\beta\gamma} = \rho_{\beta\gamma} \cdot \boxed{\exp(-\rho_\beta \tau_\beta - \rho_\gamma \tau_\gamma + \rho_{\beta\gamma} \tau_\beta)} \end{cases}$$

It is no longer applicable, which is replaced by the live-to-total time ratio T_C of coincidence channel.

② dead-time and resolving-time correction

- **For coincidence channel**

When using equal dead-time and equal resolving-time for beta and gamma channel, the Müller correction formula can be simplified as follows

$$\tau_\beta = \tau_\gamma = \tau \qquad r_\beta = r_\gamma = r$$

Müller correction formula

$$\begin{cases} R_c = R_f + R_{\beta\gamma} \\ R_f = \dfrac{e^{-\rho_{\beta\gamma}\cdot\tau}}{\rho_{\beta\gamma}} \cdot (1 - e^{-\rho_{\beta\gamma}\cdot\tau}) \cdot [\rho_\beta \cdot (\rho_\gamma - \rho_{\beta\gamma}) + \rho_\gamma \cdot (\rho_\beta - \rho_{\beta\gamma})] \cdot e^{-(\rho_\beta + \rho_\gamma - 2\rho_{\beta\gamma})\cdot\tau} \\ R_{\beta\gamma} = \rho_{\beta\gamma} \cdot e^{-(\rho_\beta + \rho_\gamma - \rho_{\beta\gamma})\cdot\tau} \end{cases}$$

It is replaced by the live-to-total time ratio T_C of coincidence channel.

③ The efficiency extrapolation method

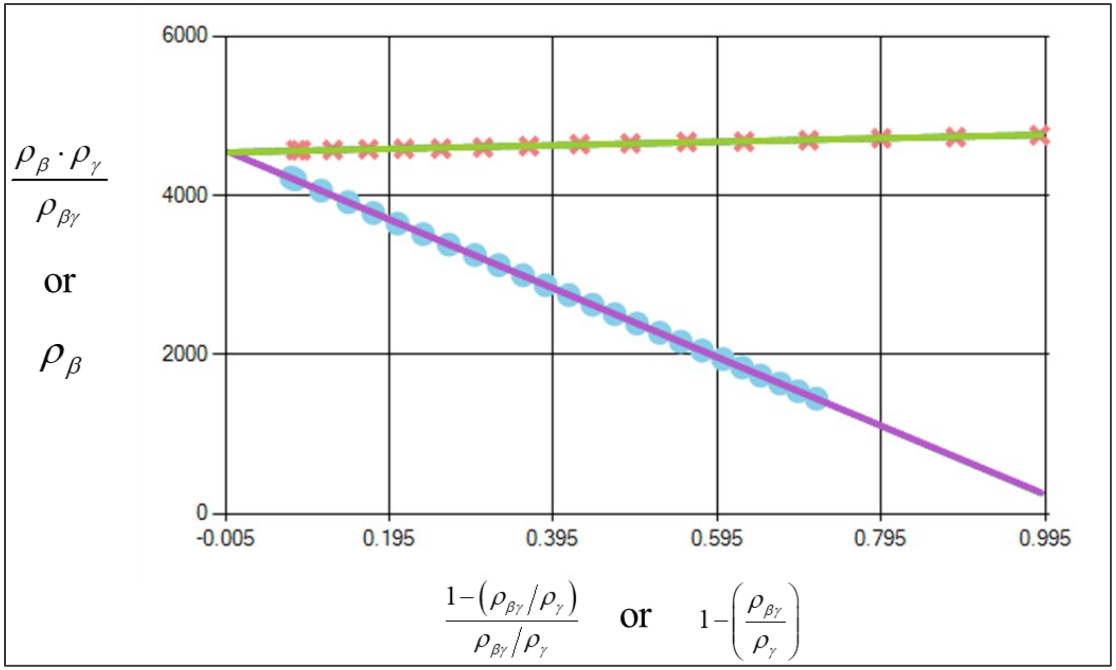
- The efficiency extrapolation method was developed to measure the γ-sensitivity in the β-channel. **Two types** of extrapolation formulas were applied.

☐ First type of extrapolation

$$\rho_\beta = A\left[1 - f_1\left[1 - \frac{\rho_{\beta\gamma}}{\rho_\gamma}\right]\right] \rightarrow A$$

☐ Second type of extrapolation

$$\frac{\rho_\beta \rho_\gamma}{\rho_{\beta\gamma}} = A\left[1 - f_2\left[\frac{1 - \rho_{\beta\gamma}/\rho_\gamma}{\rho_{\beta\gamma}/\rho_\gamma}\right]\right] \rightarrow A$$



- For each extrapolation, both linear and second order polynomial fitting were used.

4. Validation

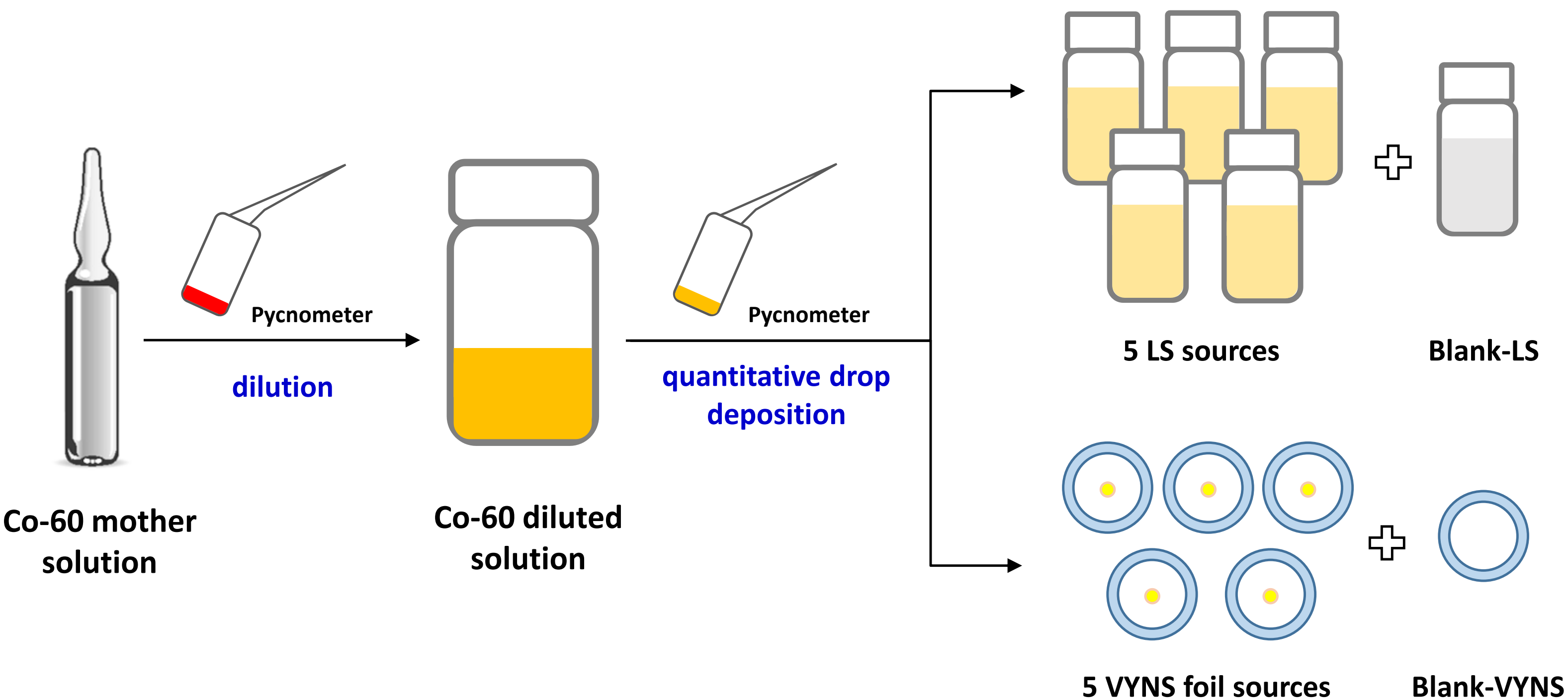
1. **Experimental validation** of Co-60 measurements at NIM

2. **Algorithm validation** by analyzing experimental List mode dataset

3. **Algorithm validation** by analyzing simulated List mode dataset

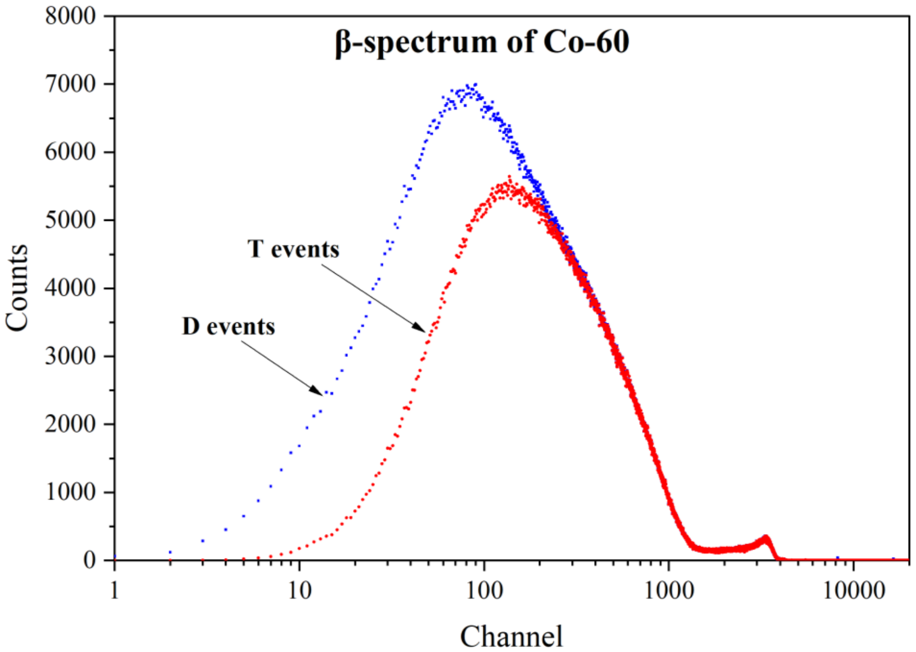
4.1 Experimental validation of Co-60 measurement at NIM

① Source preparation

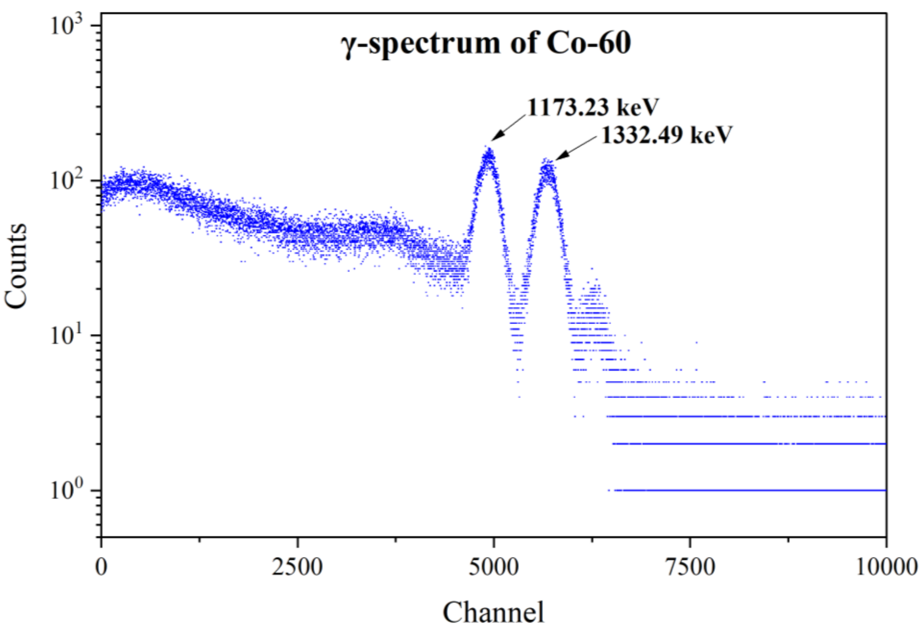


4.1 Experimental validation of Co-60 measurement at NIM

② Typical spectra measured by 4πβ(LS)-γ DCC system



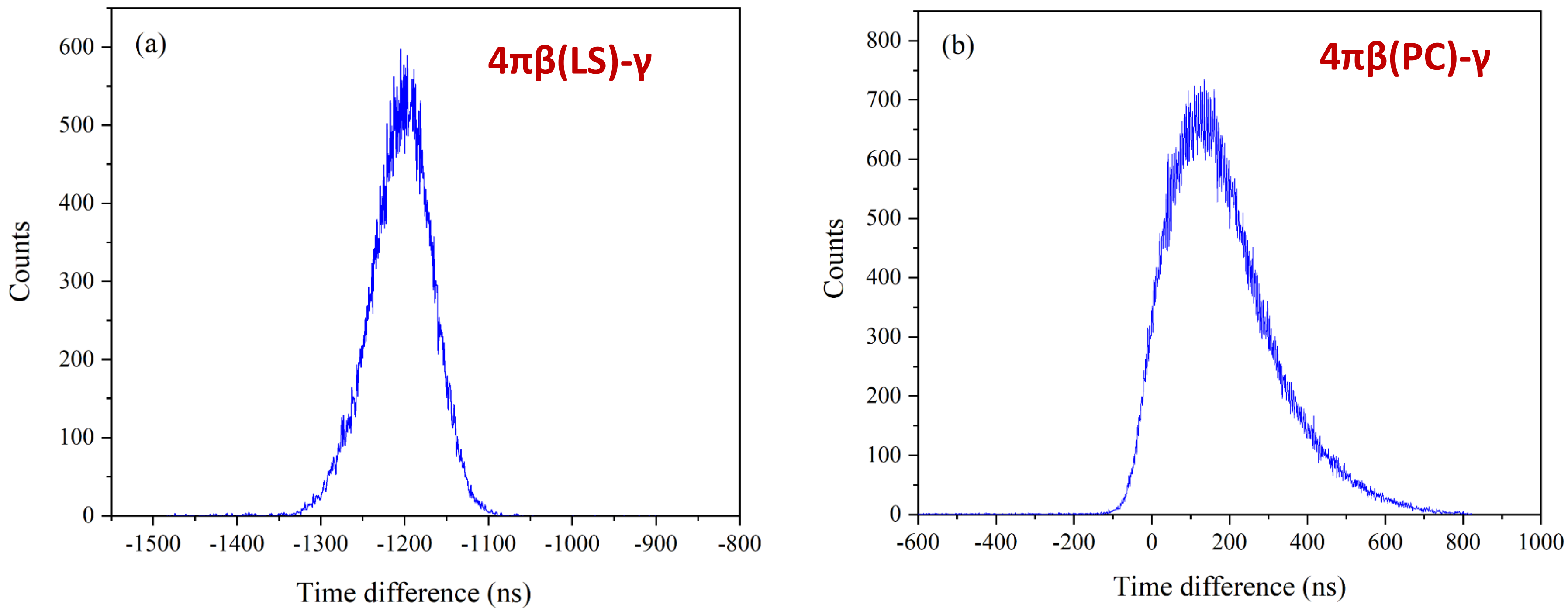
Typical beta spectrum



Typical gamma spectrum

4.1 Experimental validation of Co-60 measurement at NIM

③ Time difference distribution



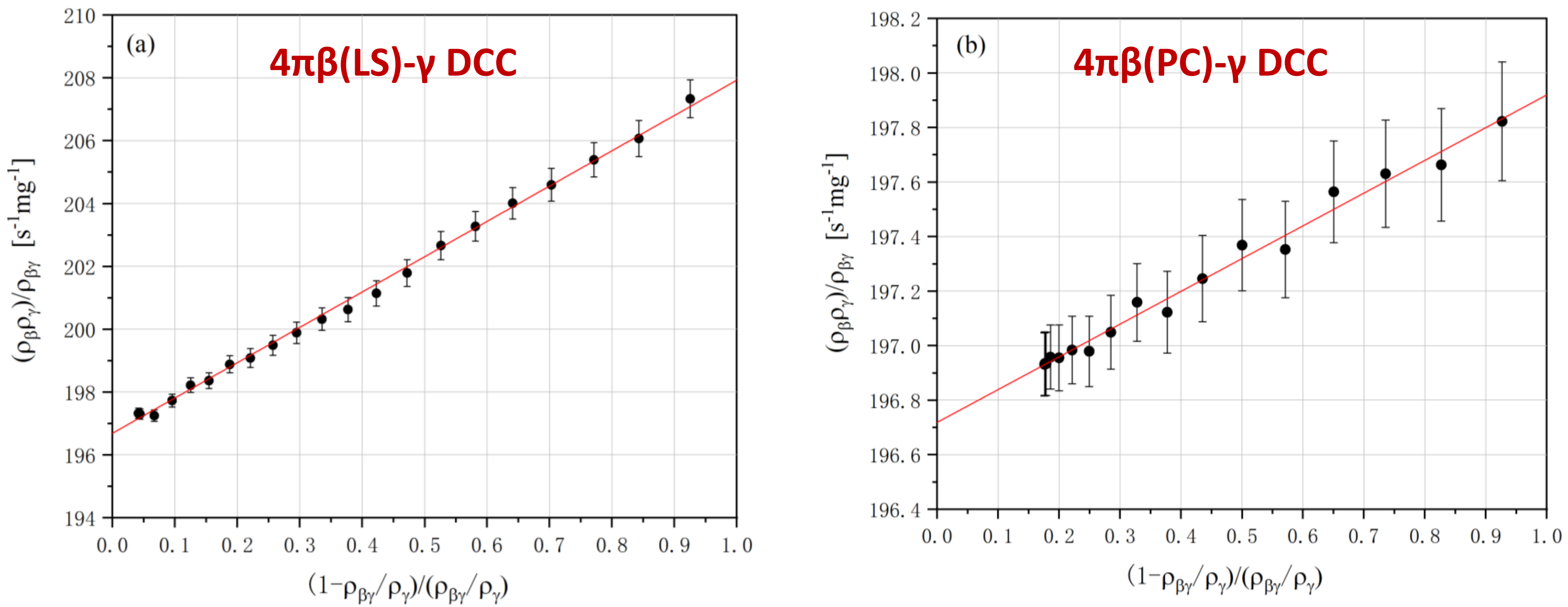
resolving-time: 200 ns

resolving-time: 800 ns

4. Validation

4.1 Experimental validation of Co-60 measurement at NIM

④ Extrapolation and γ-sensitivity in the β-channel

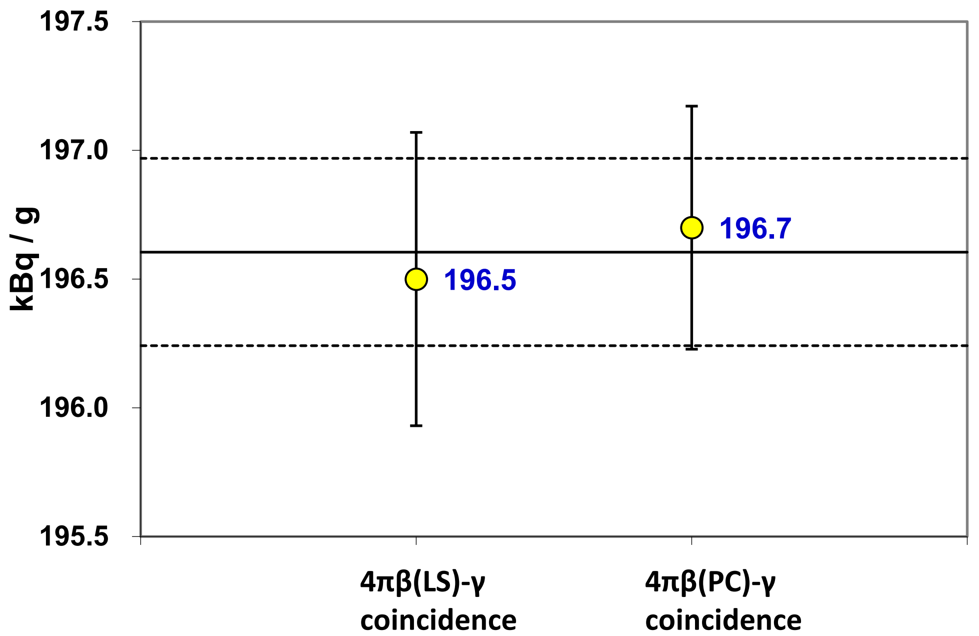


Method	Slope of fit line	$\varepsilon_{\beta\gamma}$
4πβ(LS)-γ DCC	11.24 kBq/g	5.8%
4πβ(PC)-γ DCC	1.20 kBq/g	0.6%

4.1 Experimental validation of Co-60 measurement at NIM

⑤ Result of the validation

Method	*A*(kBq/g)	*u*(*A*)/*A*
4πβ(LS)-γ coincidence counting	196.51	0.29%
4πβ(PC)-γ coincidence counting	196.71	0.24%



The results are consistent within uncertainty, **with relative deviation of -0.10%.**

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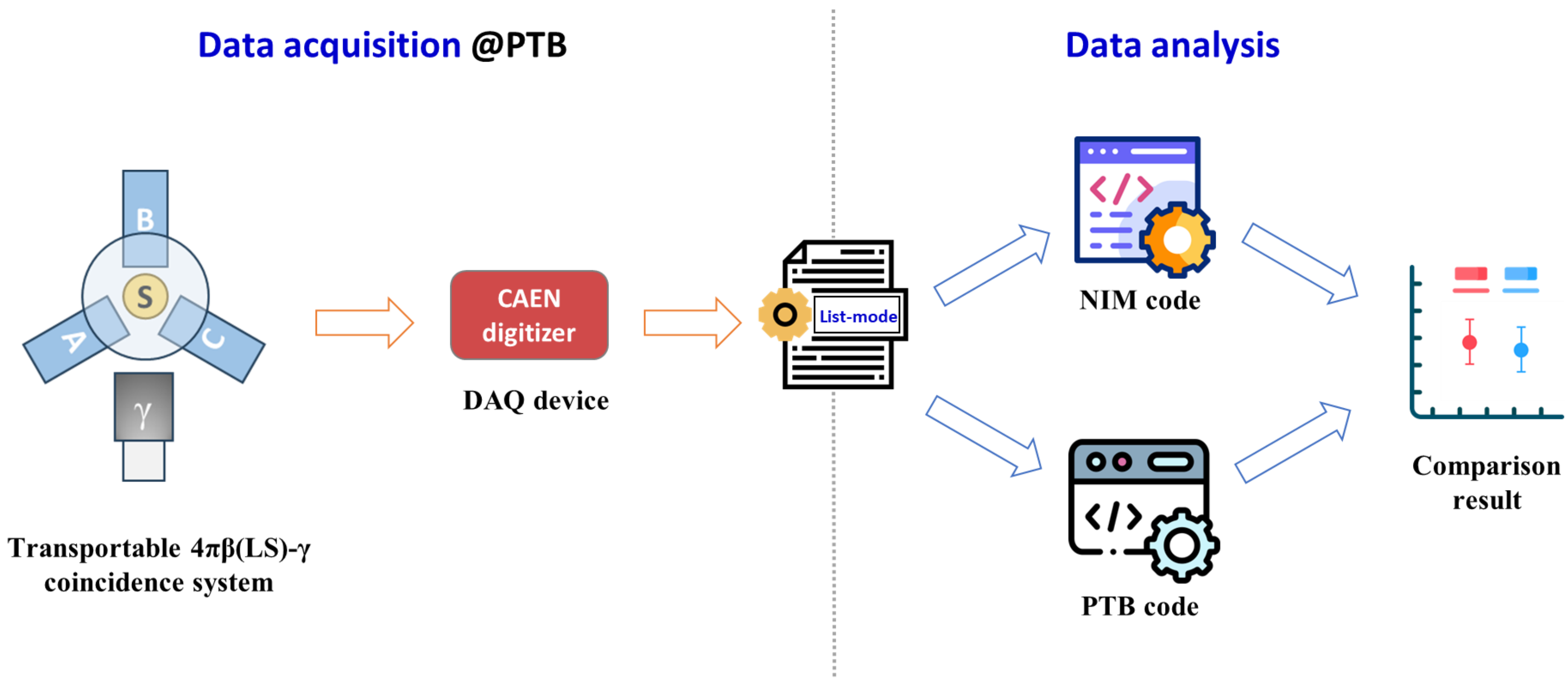
4.2 Algorithm validation by analyzing experimental List mode dataset

① Overview of the algorithm validation between PTB and NIM



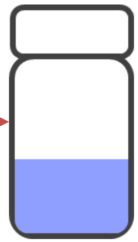
4.2 Algorithm validation by analyzing experimental List mode dataset

② Data acquisition

@ PTB

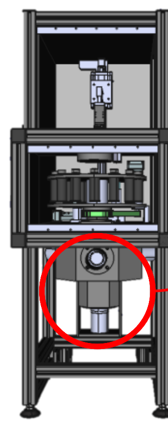
Data acquisition

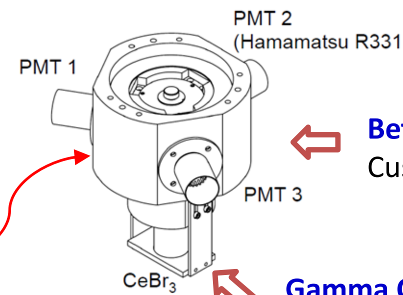
Source: Co-60



- The comparison was based on the same experimental dataset for Co-60, acquired by PTB's 4πβ(LS)-γ coincidence counting system.

Detection setup: 4πβ(LS)-γ coincidence system at PTB





Beta Channel: Custom-built TDCR counter

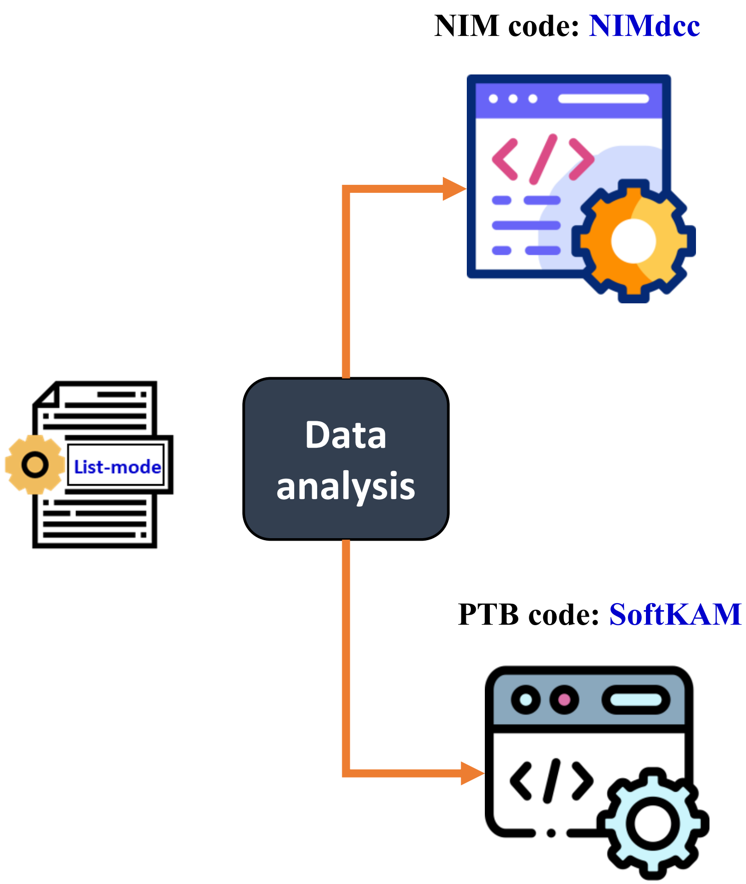
Gamma Channel: CeBr3 scintillation detector

Data acquisition device: CAEN N6751C digitizer

- The data acquisition was performed by CAEN N6751C digitizer, and registered events from each detector were saved in list-mode data format.

4.2 Algorithm validation by analyzing experimental List mode dataset

③ Data analysis



- **Similarity**

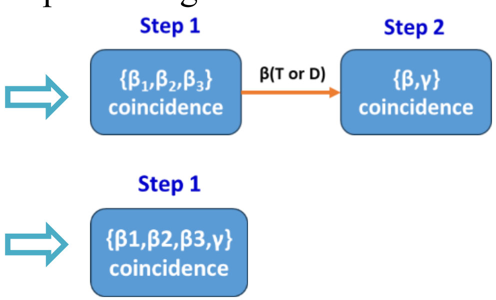
Both codes include several similar functions: Spectrum generation, signal processing, count rate correction, efficiency extrapolation, etc.

- **Difference**

The codes of NIM and PTB adopt very different strategies for coincidence and dead-time processing.

NIM code uses a two-step coincidence approach

PTB code applies a one-step coincidence approach



The SoftKAM approach counting rates for D-G and T-G coincidences are obtained in the same evaluation step, because of the use of common dead-time

4.2 Algorithm validation by analyzing experimental List mode dataset

③ Data analysis - Count rate correction

SoftKAM

Campion formula

- live time correction

NIMdcc

Müller formula

- live time correction

$$\rho_{\beta\gamma} = \frac{R_c - (r_\beta + r_\gamma) R_\beta R_\gamma}{(1 - R_\beta \tau_\beta - R_\gamma \tau_\gamma + R_c \tau_m)(1 - R_\gamma r_\beta - R_\beta r_\gamma)}$$

replaced by live-to-real time ratio $T_{L,c}$

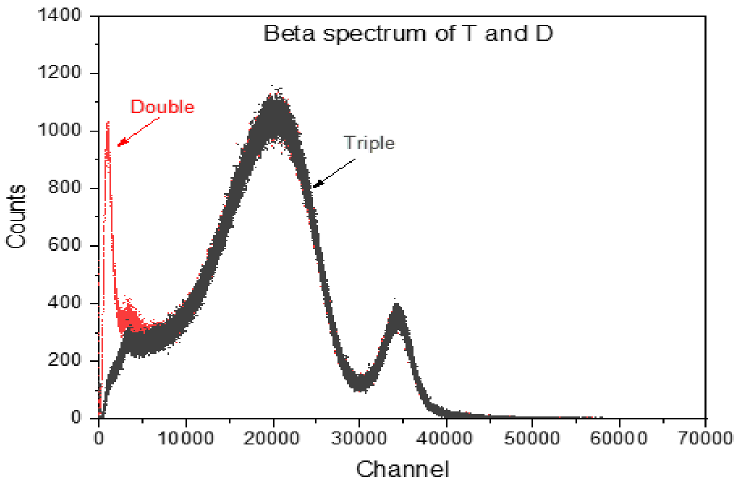
$$\rho_{\beta\gamma} = \frac{R_c - (r_\beta + r_\gamma) R_\beta R_\gamma}{T_{L,c} \cdot (1 - R_\gamma r_\beta - R_\beta r_\gamma)}$$

$$\begin{cases} R_c = R_f + R_{\beta\gamma} \\ R_f = \dfrac{R_\beta \rho_g}{\rho_{\beta\gamma}} \left[\exp(-\rho_\gamma \tau_\gamma + \rho_{\beta\gamma} \tau_\beta)\right] \cdot \left[1 - \exp(-\rho_{\beta\gamma} r_\gamma)\right] \\ \quad + \dfrac{R_\gamma \rho_b}{\rho_{\beta\gamma}} \left[\exp(-\rho_b \tau_\beta)\right] \cdot \begin{cases} \rho_{\beta\gamma} \left[r_\beta - \left|r_\beta - \tau_\gamma + \tau_\beta\right|\right] \\ +1 - \exp\left(-\rho_{\beta\gamma} \left|r_\beta - \tau_\gamma + \tau_\beta\right|\right) \end{cases} \\ R_{\beta\gamma} = \rho_{\beta\gamma} \cdot \boxed{\exp(-\rho_\beta \tau_\beta - \rho_\gamma \tau_\gamma + \rho_{\beta\gamma} \tau_\beta)} \end{cases}$$

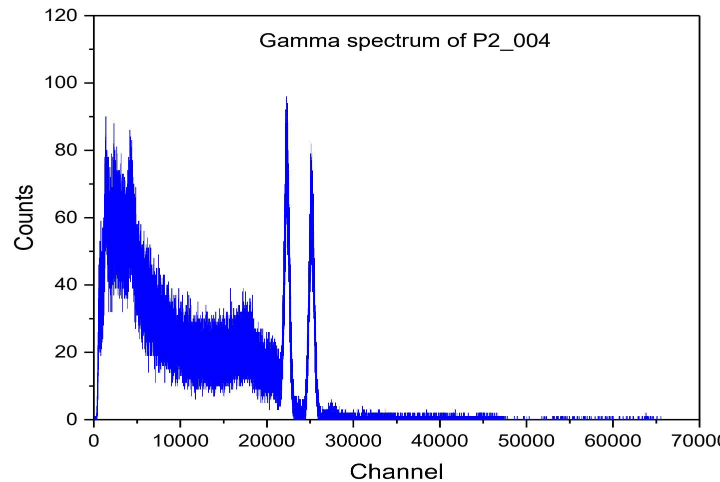
It is no longer applicable, replaced by live-to-real time ratio $T_{L,c}$

4.2 Algorithm validation by analyzing experimental List mode dataset

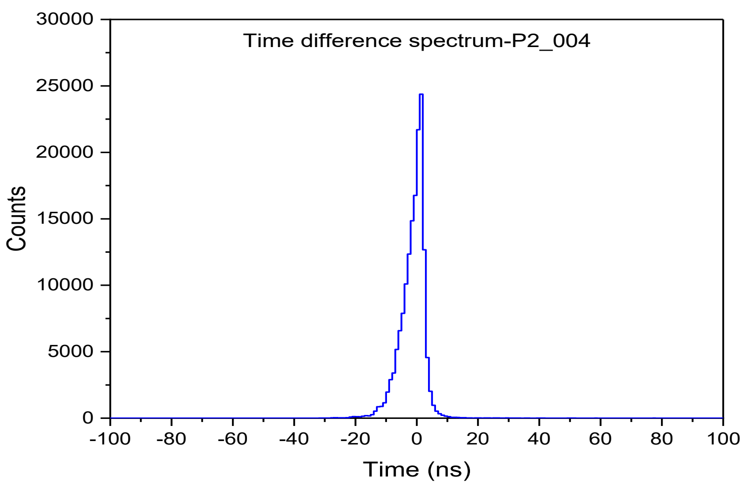
④ Results: typical spectrum



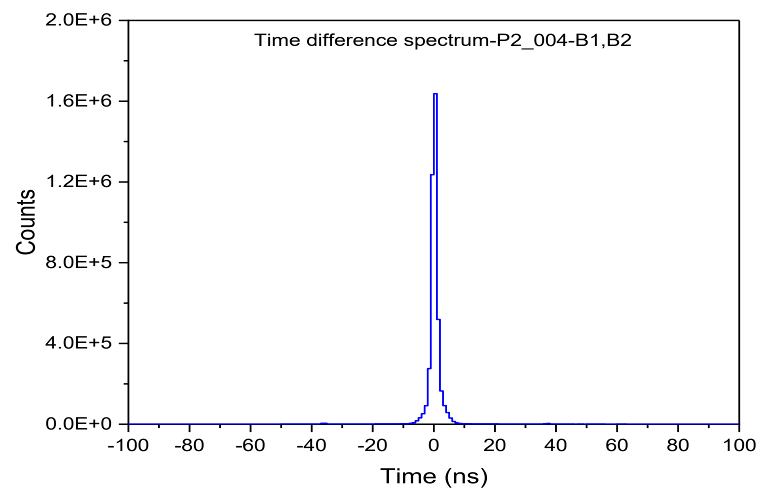
Beta spectrum



Gamma spectrum



Time difference spectrum between β_1-β_2



Time difference spectrum between β-γ

4.2 Algorithm validation by analyzing experimental List mode dataset

④ Data format conversion

Row	Channel	TimeStamp	Elong	Eshort	PU	Extra
0	3	233000	20681	16204	0	0
1	1	233000	28253	18096	0	0
2	2	233001	22577	18202	0	0
3	3	233431	1145	1174	0	0
4	1	250195	22293	19725	0	0
5	3	250198	15822	15415	0	0
6	2	250198	16217	14665	0	0
7	3	491146	599	591	0	0
8	3	528238	11030	10051	0	0
9	2	528238	14155	13861	0	0
10	1	528238	13811	13723	0	0
11	3	609340	1573	1596	0	0
12	1	609354	841	845	0	0
13	3	675839	27832	23469	0	0
14	2	675840	28105	23586	0	0
15	1	675842	23412	21679	0	0
16	2	774048	19019	17614	0	0
17	1	774048	17855	17047	0	0
18	3	774049	17056	16571	0	0
19	3	774362	1091	1125	0	0
20	1	778832	2635	2654	0	0

DBJ00035_2310-02_P1_012_0.csv

文件 编辑 查看

Time unit conversion

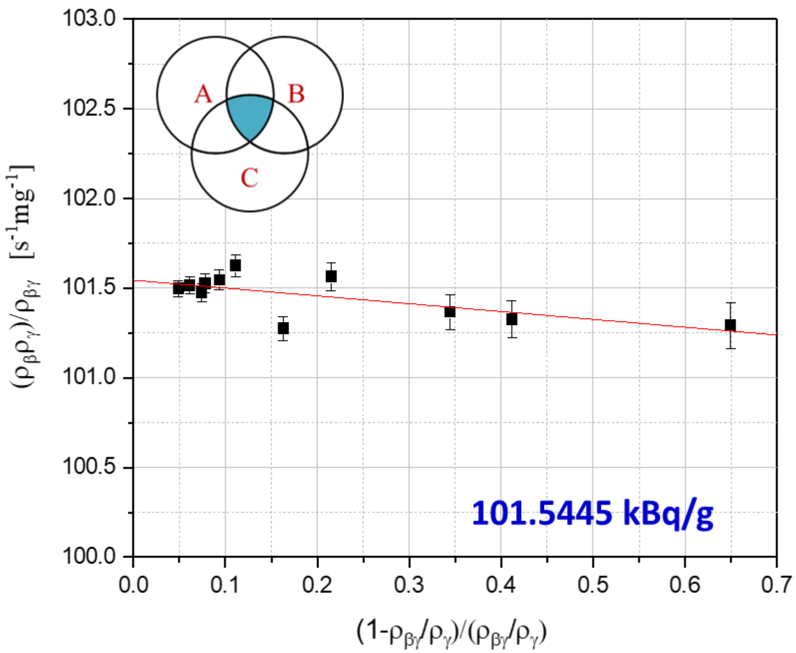
```
BOARD;CHANNEL;TIMETAG;ENERGY;ENERGYSHORT;FLAGS
0;0;20787595000;6691;0;0
0;0;41245420000;2564;0;0
0;0;43359049000;1473;0;0
0;0;89164717000;7659;0;0
0;0;226153439000;3894;0;0
0;0;240289818000;4619;0;0
0;0;338555458000;13066;0;0
0;0;357038327000;3784;0;0
0;0;388174872000;2891;0;0
0;0;429556479000;3872;0;0
0;0;445102358000;3049;0;0
0;0;511708610000;13513;0;0
0;0;516524353000;3023;0;0
0;0;595504506000;5250;0;0
```

PTB data format **NIM data format**

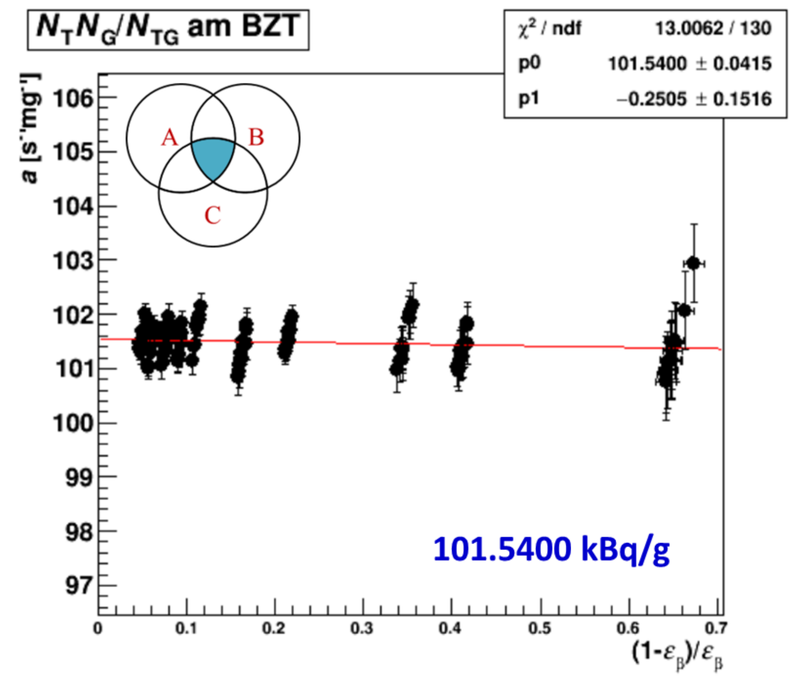
30

4.2 Algorithm validation by analyzing experimental List mode dataset

⑤ Results: T-G coincidence



T-G extrapolation result of NIM

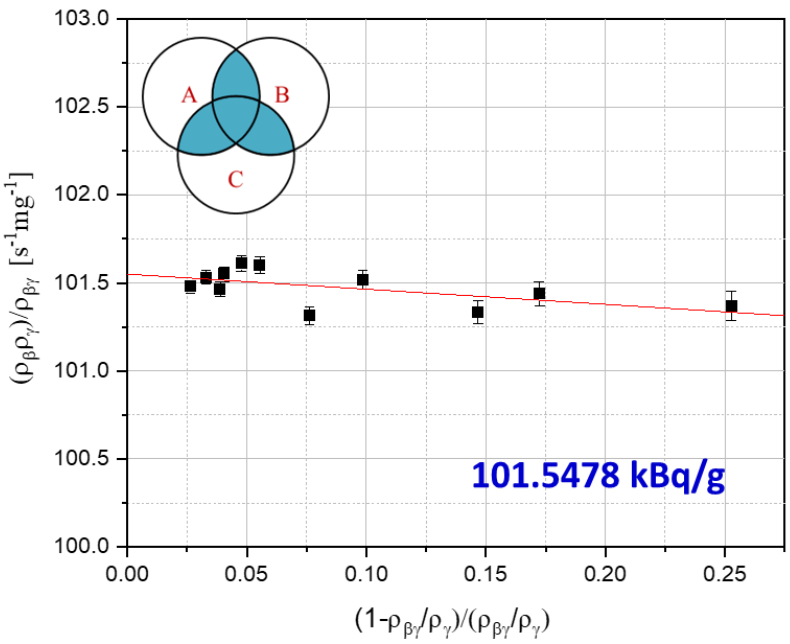


T-G extrapolation result of PTB

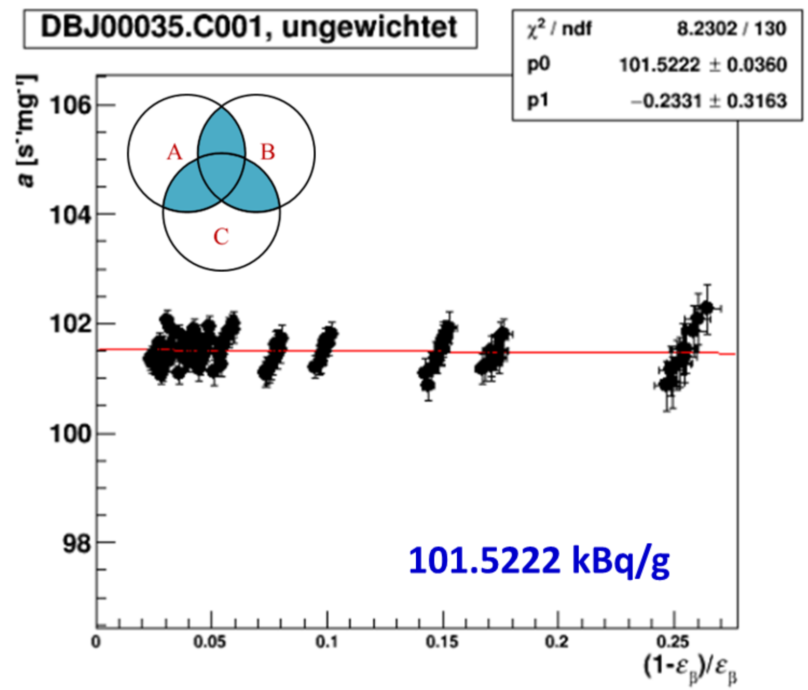
The relative deviation of T-G coincidence results between PTB and NIM is only about **0.004%.**

☐ 4.2 Algorithm validation by analyzing experimental List mode dataset

⑤ **Results: D-G coincidence**



D-G extrapolation result of NIM



D-G extrapolation result of PTB

And the relative deviation of D-G coincidence results between PTB and NIM is only about **0.03%.**

4.2 Algorithm validation by analyzing experimental List mode dataset

⑤ **Results: summary**

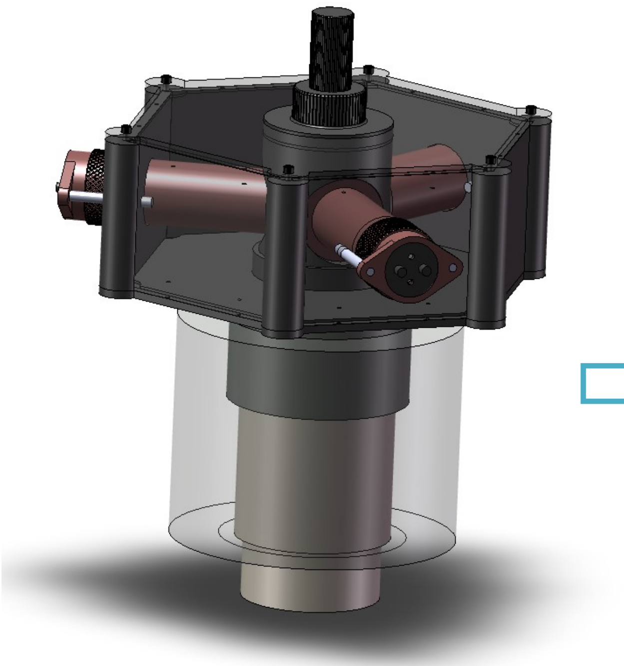
Institution	Extrapolation mode	Activity (kBq/g)
NIM	T-G	101.545
	D-G	101.548
PTB	T-G	101.540
	D-G	101.522

- The analysis results for the NIM and PTB codes were in excellent agreement.
- The maximum relative deviation for all four results does not exceed 0.03%.

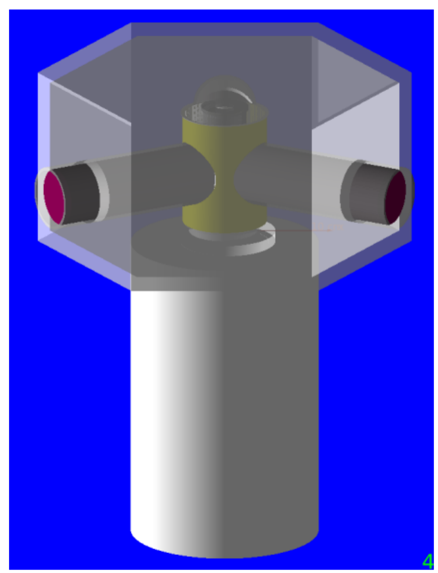
4.3 Algorithm validation by analyzing simulated List mode dataset

① Generation of simulated list-mode digital dataset

- Geant4 was chosen to generate the list-mode digital datasets for its capability to model radionuclides as sources and simulate particle emissions from related cascade with time correlations across transitions and de-excitation processes, and its flexibility in adjusting the output of required data and format.
- The geometric parameters of the model was referenced from the 4πβ(LS)-γ systems at NIM



4πβ(LS)-γ systems at NIM



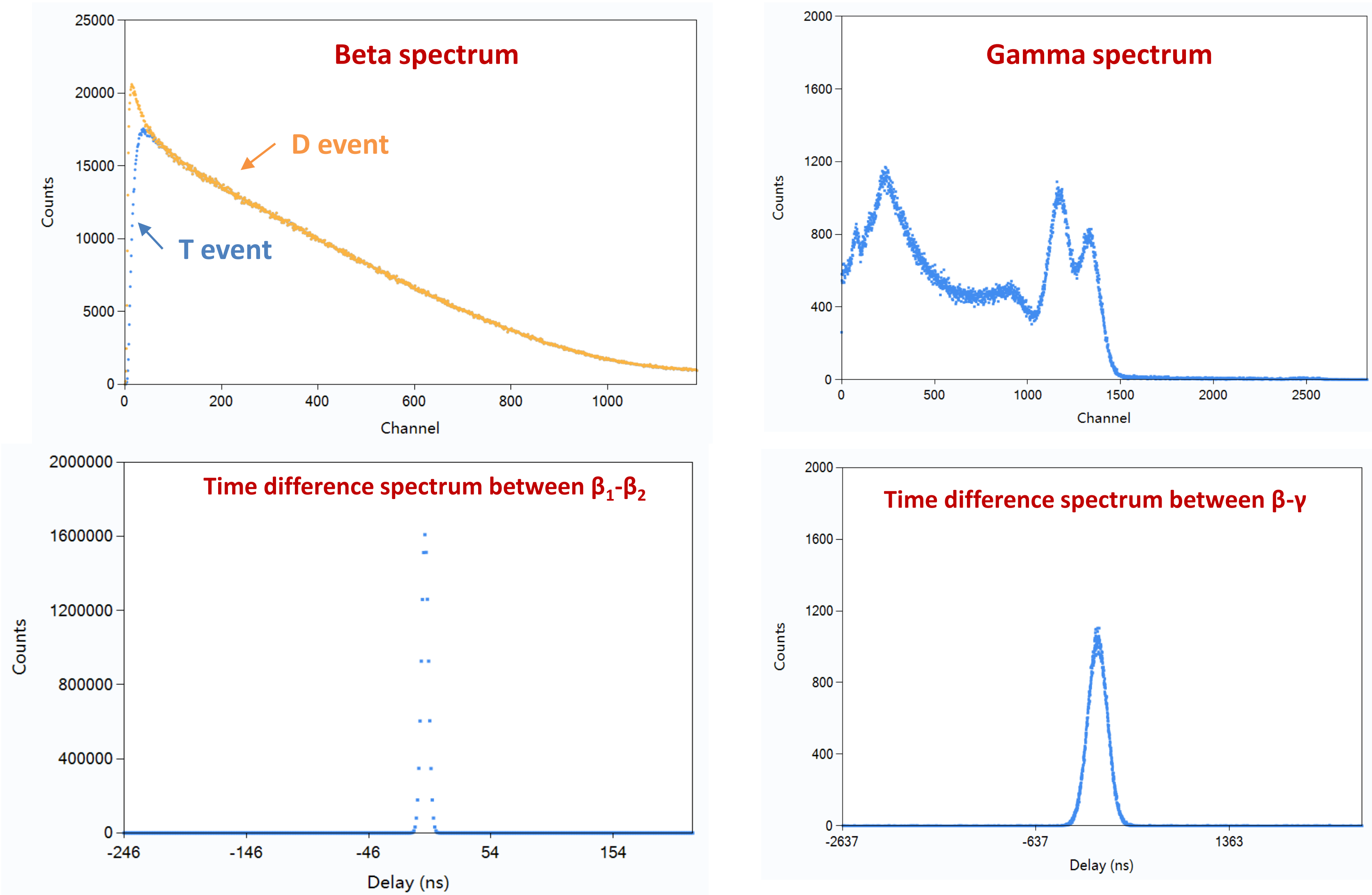
Geant4 model

beta_0.csv
beta_1.csv
beta_2.csv
gamma_0.csv

Simulated List mode dataset

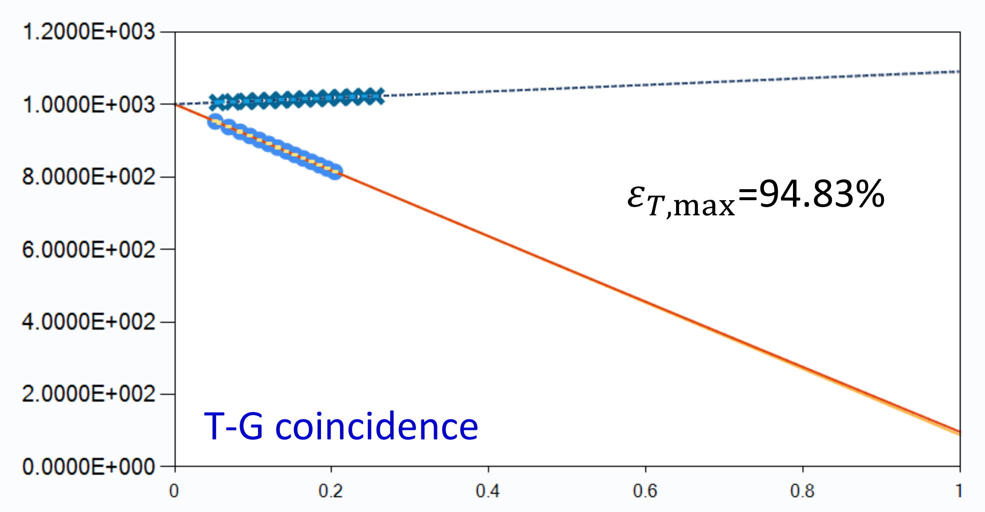
◻ 4.3 Algorithm validation by analyzing simulated List mode dataset

② Typical spectra



4.3 Algorithm validation by analyzing simulated List mode dataset

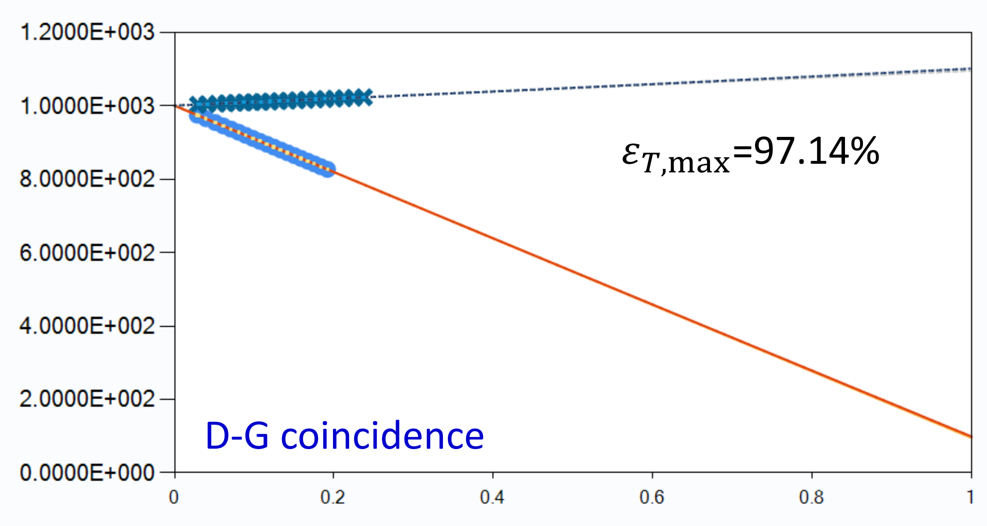
② Results: T-G and D-G coincidence



True Activity: 1000.05 Bq

T-G coin: 1000.95 Bq

Relative deviation: 0.09%



True Activity: 1000.05 Bq

T-G coin: 1000.76 Bq

Relative deviation: 0.07%

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Thanks
for your attention